### *MAXIMUM/MINIMUM PROBLEMS*

The following problems are maximum/minimum optimization problems. They illustrate one of the most important applications of the first derivative. Many students find these problems intimidating because they are "word" problems, and because there does not appear to be a pattern to these problems. However, if you are patient you can minimize your anxiety and maximize your success with these problems by following these guidelines :

GUIDELINES FOR SOLVING MAX./MIN. PROBLEMS

1. Read each problem slowly and carefully. Read the problem at least three times before trying to solve it. Sometimes words can be ambiguous. It is imperative to know exactly what the problem is asking. If you misread the problem or hurry through it, you have NO chance of solving it correctly.

2. If appropriate, draw a sketch or diagram of the problem to be solved. Pictures are a great help in organizing and sorting out your thoughts.

3. Define variables to be used and carefully label your picture or diagram with these variables. This step is very important because it leads directly or indirectly to the creation of mathematical equations.

4. Write down all equations which are related to your problem or diagram. Clearly denote that equation which you are asked to maximize or minimize. Experience will show you that MOST optimization problems will begin with two equations. One equation is a "constraint" equation and the other is the "optimization" equation. The "constraint" equation is used to solve for one of the variables. This is then substituted into the "optimization" equation before differentiation occurs. Some problems may have NO constraint equation. Some problems may have two or more constraint equations.

5. Before differentiating, make sure that the optimization equation is a function of only one variable. Then differentiate using the well-known rules of differentiation.

6. Verify that your result is a maximum or minimum value using the first or second derivative test for extrema.

The following problems range in difficulty from average to challenging.

*PROBLEM 1 :* Find two nonnegative numbers whose sum is 9 and so that the product of one number and the square of the other number is a maximum.

*PROBLEM 2 :* Build a rectangular pen with three parallel partitions using 500 feet of fencing. What dimensions will maximize the total area of the pen ?

*PROBLEM 3 :* An open rectangular box with square base is to be made from 48 ft.2 of material. What dimensions will result in a box with the largest possible volume ?

*PROBLEM 4 :* A container in the shape of a right circular cylinder with no top has surface area 3 ft.2 What height h and base radius r will maximize the volume of the cylinder ?

*PROBLEM 5 :* A sheet of cardboard 3 ft. by 4 ft. will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with largest volume ?

*PROBLEM 6 :* Consider all triangles formed by lines passing through the point (8/9, 3) and both the x- and y-axes. Find the dimensions of the triangle with the shortest hypotenuse.

*PROBLEM 7 :* Find the point (x, y) on the graph of nearest the point (4, 0).

*PROBLEM 8 :* A cylindrical can is to hold 20 m.3 The material for the top and bottom costs $10/m.2 and material for the side costs $8/m.2 Find the radius r and height h of the most economical can.

*PROBLEM 9 :* You are standing at the edge of a slow-moving river which is one mile wide and wish to return to your campground on the opposite side of the river. You can swim at 2 mph and walk at 3 mph. You must first swim across the river to any point on the opposite bank. From there walk to the campground, which is one mile from the point directly across the river from where you start your swim. What route will take the least amount of time ?

*PROBLEM 10 :* Construct a window in the shape of a semi-circle over a rectangle. If the distance around the outside of the window is 12 feet, what dimensions will result in the rectangle having largest possible area ?

*PROBLEM 11 :* There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of trees ?

*PROBLEM 12 :* Find the dimensions of the rectangle of largest area which can be inscribed in the closed region bounded by the x-axis, y-axis, and graph of *y*=8-*x*3 . (See diagram.)

*PROBLEM 13 :* Consider a rectangle of perimeter 12 inches. Form a cylinder by revolving this rectangle about one of its edges. What dimensions of the rectangle will result in a cylinder of maximum volume ?

*PROBLEM 14 :* A movie screen on a wall is 20 feet high and 10 feet above the floor. At what distance x from the front of the room should you position yourself so that the viewing angle of the movie screen is as large as possible ? (See diagram.)



*PROBLEM 15 :* Find the dimensions (radius r and height h) of the cone of maximum volume which can be inscribed in a sphere of radius 2.

*PROBLEM 16 :* What angle between two edges of length 3 will result in an isosceles triangle with the largest area ? (See diagram.)



*PROBLEM 17 :* Of all lines tangent to the graph of , find the tangent lines of mimimum slope and maximum slope.

*PROBLEM 18 :* Find the length of the shortest ladder that will reach over an 8-ft. high fence to a large wall which is 3 ft. behind the fence. (See diagram.)



*PROBLEM 19 :* Find the point P = (x, 0) on the x-axis which minimizes the sum of the squares of the distances from P to (0, 0) and from P to (3, 2).

*PROBLEM 20 :* Car B is 30 miles directly east of Car A and begins moving west at 90 mph. At the same moment car A begins moving north at 60 mph. What will be the minimum distance between the cars and at what time t does the minimum distance occur ?

*PROBLEM 21 :* A rectangular piece of paper is 12 inches high and six inches wide. The lower right-hand corner is folded over so as to reach the leftmost edge of the paper (See diagram.).



Find the minimum length of the resulting crease.

http://www.ma.utexas.edu/cgi-pub/kawasaki/plain/derAppl/3.html

1. Find the dimensions of the rectangle of perimeter 120cm that has the largest area.

2. Find the coordinates of P that maximize the area of the rectangle shown in the figure below.

3. The base of a triangle is on the x-axis, one side lies along the line y = 3x, and the third side passes through the point (1,1). What is the slope of the third side if the area of the triangle is to be a minimum?

4. A triangle is formed by the coordinate axes and a line through the point (2,5). Determine the slope of this line if the area of the triangle is to be a minimum.

**Rescue!!**

The sky is blue, the sun is hot......you are lying on the beach, almost asleep, when suddenly you become aware of a commotion. Someone is in trouble in the water! Since there is no-one else around and you have recently obtained your life-saving award, it is up to you to go to the rescue. But what is the fastest way of reaching them?

Quickly you review the situation:

* they are 80m along the beach from you and 60m from the shore
* you can swim at 2 m/s
* you can run at 5 m/s.

**How can you reach them in the shortest possible time?**